Lecture Notes on **Discrete Mathematics**.
Birzeit University, Palestine, 2020

First Order Logic

Mustafa Jarrar

3.1 Introduction to First Order Logic

3.2 Negation and Conditional Statements

3.3 Multiple Quantifiers in First Order Logic

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 3 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

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First Order Logic

3.2 Conditional Statements

In this lecture:

- ❑ Part 1: **Negations of Quantified Statements**
- ❑ Part 2: Contrapositive, Converse, and Inverse
- ❑ Part 3: Necessary, Sufficient, Iff

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Negations of Quantified Statements

How to negate these statement:

All Palestinians like Zatar

Some Palestinians do not like Zatar

Some Palestinians Like Zatar

All Palestinians do not like Zatar

Theorem 3.2.2 Negation of an Existential Statement

The negation of a statement of the form

$$\forall x \text{ in } D,\ Q(x)$$

is logically equivalent to a statement of the form

$$\exists x \text{ in } D \quad \text{such that not } Q(x)$$

Symbolically, $\sim[\exists x \in D\ .\ Q(x)] \equiv \forall x \in D\ .\ \sim Q(x)$

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Negations of Quantified Statements

$\forall\, p \in \text{Prime} \,.\, \text{Odd}(p)$

$\exists p \in \text{Prime} \,.\, {\sim}\text{Odd}(p)$

Some computer hackers are over 40

All computer hackers are not over 40

All computer programs are finite

Some computer programs are not finite

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Negations of Quantified Statements

No politicians are honest

Some politicians are honest

$\forall x \,.\, P(x) \rightarrow Q(x)$

$\exists x \,.\, P(x) \wedge {\sim}Q(x)$

$\forall p \in \text{Person} \,.\, \text{Blond}(p) \rightarrow \text{BlueEyes}(p)$

$\exists p \in \text{Person} \,.\, \text{Blond}(p) \wedge {\sim}\text{BlueEyes}(p)$

If a computer program has more than 10000 lines then it contains a bug

a computer program has more than 10000 and does not contains a bug

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3.2 Conditional Statements

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- Part 1: Negations of Quantified Statements
- Part 2: **Contrapositive, Converse, and Inverse**
- Part 3: Necessary and Sufficient Conditions, Only If

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Variants of Universal Conditional Statements

Definition

Consider a statement of the form: $\forall x \in D \;.\; P(x) \rightarrow Q(x)$.

1. Its **contrapositive** is the statement: $\forall x \in D \;.\; \sim Q(x) \rightarrow \sim P(x)$
2. Its **converse** is the statement: $\forall x \in D \;.\; Q(x) \rightarrow P(x)$.
3. Its **inverse** is the statement: $\forall x \in D \;.\; \sim P(x) \rightarrow \sim Q(x)$.

Example: $\forall x \in \text{Person} \;.\; \text{Palestinian}(x) \rightarrow \text{Smart}(x)$

Contrapositive: $\forall x \in \text{Person} \;.\; \sim \text{Smart}(x) \rightarrow \sim \text{Palestinian}(x)$

Converse: $\forall x \in \text{Person} \;.\; \text{Smart}(x) \rightarrow \text{Palestinian}(x)$

Inverse: $\forall x \in \text{Person} \;.\; \sim \text{Palestinian}(x) \rightarrow \sim \text{Smart}(x)$

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Variants of Universal Conditional Statements

$$\forall x \in \mathbf{R} \ . \ \text{MoreThan}(x,2) \rightarrow \text{MoreThan}(x^2,4)$$

$$\forall x \in \mathbf{R} \ . \ x > 2 \ \rightarrow \ x^2 > 4$$

Contrapositive: $\forall x \in \mathbf{R} \ . \ x^2 \leq 4 \rightarrow x \leq 2$

Converse: $\forall x \in \mathbf{R} \ . \ x^2 > 4 \rightarrow x > 2$

Inverse: $\forall x \in \mathbf{R} \ . \ x \leq 2 \ \rightarrow \ x^2 \leq 4$

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- ❑ Part 3: **Necessary and Sufficient Conditions, Only If**

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Necessary and Sufficient Conditions

Definition

"$\forall x \,.\, r(x)$ is a **sufficient condition** for $s(x)$" means "$\forall x \,.\, r(x) \rightarrow s(x)$"

"$\forall x \,.\, r(x)$ is a **necessary condition** for $s(x)$" means "$\forall x, \sim r(x) \rightarrow \sim s(x)$"
or, equivalently, "$\forall x, s(x) \rightarrow r(x)$"

Examples

Squareness is a sufficient condition for rectangularity.

If something is a square, then it is a rectangle.

$\forall x$. Square(x) → Rectangular(x)

To get a job it is sufficient to be loyal.

If one is loyal (s)he will get a job

$\forall x$. Loyal(x) → GotaJob(x)

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Examples

Being smart is necessary to get a job.

If you are not smart you don't get a job

If you got a job then you are smart

$\forall x$. ~Smart(x) → ~GotaJob(x)

$\forall x$. GotaJob(x) → Smart(x)

Being above 40 years is necessary for being president of Palestine

$\forall x$. ~Above(x, 40) → ~CanBePresidentOfPalestine(x)

$\forall x$. CanBePresidentOfPalestine(x) → Above(x, 40)

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محذوف

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or, equivalently, "$\forall x, s(x) \rightarrow r(x)$"

"$\forall x \,.\, r(x)$ **only if** $s(x)$" means "$\forall x, \sim s(x) \rightarrow \sim r(x)$"
or, equivalently, "$\forall x$, if $r(x)$ then $s(x)$"

Examples

You get the job only if you are the top.

If you are not a top you will not get a job

If you got the job then you a top

$\forall x \,.\, \sim Top(x) \rightarrow \sim GotaJob(x)$

$\forall x \,.\, GotaJob(x) \rightarrow Top(x)$

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